

## Determination of Plasticity for Pre-Deformed Billet

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**Abstract.** The calculation procedure for determining the plasticity of pre-deformed metals during their processing by pressure has been developed. The calculation procedure is based on a fracture model, which in turn is based on the tensor description of damage accumulation. With known mechanical characteristics, as well as with known plasticity diagrams, the fracture model makes it possible to evaluate the plasticity of pre-deformed bend for any kind of stress state. When manufacturing steeply curved branches using the pipe extrusion method, the procedure was tested. Verification of the mathematical model has shown a high level of its adequacy, and it can be used in assessing the plasticity of pre-deformed billet.

### Introduction

Development and industrial introduction of effective metal forming technologies, which ensure a high level of operational properties and reliability of products is one of the most important tasks of engineering.

After various operations of metal forming, a technological heritage is formed. These are residual stresses, hardening, deformations gradient, residual plasticity, and other factors. These factors influence in the future the performance of products, which predetermines the task of creating methods for their evaluation. Most of these factors have been sufficiently investigated [1].

### Analysis of Last Research and Publications

Research of the mechanical properties anisotropy and the plasticity resource, after metal plastic deformation by its plasticity during next plastic deformation are presented in [2]. In this case, a tensor model of damage accumulation was used. It is based on the hypothesis that the intensity of metal damage accumulation depends on the plasticity sensitivity to the stress state scheme.

In [3], a method for determining the plasticity of metallic materials was proposed. It is implemented using the dynamic indentation of the material with a spherical tip. The measured value of plasticity was determined by the ratio of plastic and total deformations in the hole formed during indentation and takes into account the effect of the material elasticity modulus.

The problems of billet deformability under a volumetric stress state were solved in [4]. The authors proposed the theory of deformability, the calculation method and created a model for metals destruction during their shape-breaking. But evaluation of the plasticity of a pre-deformed billet, is a difficult and insufficiently investigated problem.

### Formulation of the Task

The purpose of this research is to develop a methodology for evaluating the plasticity of a pre-deformed metal.

## Research Results

Accumulated at all stages of deformation, the strain intensity (parameter Udquist), which is called the ultimate deformation ( $\varepsilon_p$ ), is taken as a measure of plasticity at the moment of destruction of the workpiece material at the place of final deformations:

$$\varepsilon_p = \int_0^{\tau_p} \dot{\varepsilon}_i d\tau, \quad (1)$$

where  $\dot{\varepsilon}_i$  – strain rates intensity.

Processes of metal forming are based on the ability of metals under the action of the applied load to pass into the plastic state.

Plasticity of metals depends on many factors, among which, in addition to the type of material, the main thermomechanical parameters of the process are: temperature, strain rate, strain state, deformation story, strain gradient and other.

The dependence of plasticity on the type of stress state with simple deformation and fixed temperature and rate process parameters is its mechanical characteristic. To determine the mechanical characteristic diagram, a material test is performed at various stress states under simple loading conditions, when the components of the stress tensor vary in proportion to one parameter.

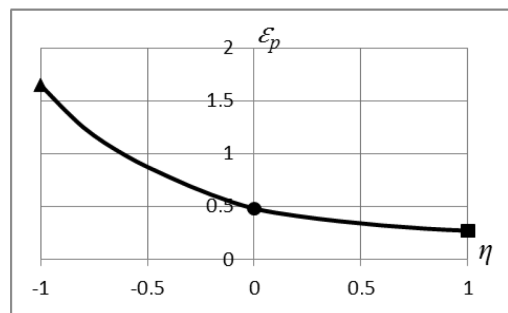
The stress state will be characterized by the indices of the stressed state. The stress state index according to G.A. Smirnov-Alyaev [5]:

$$\eta = \frac{I_1(T_\sigma)}{\sqrt{3}I_2(D_\sigma)} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sigma_i}, \quad (2)$$

where  $I_1(T_\sigma)$  – the first invariant of the stress tensor,  $\sigma_1, \sigma_2, \sigma_3$  are the principal stresses,  $I_2(D_\sigma)$  is the second invariant of the stress deviator or stress intensity:

$$\sigma_i = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}. \quad (3)$$

The parameter  $\eta$  is useful when using the plasticity diagrams in the coordinates  $\varepsilon_p = f(\eta)$ , and is equal to:  $\eta = 1$  – uniaxial tension,  $\eta = -1$  – uniaxial compression,  $\eta = 0$  – shift (Fig. 1).



▲ – compression, ● – torsion, ■ – stretching.

Fig. 1. The plasticity diagram of steel 20

Plasticity diagrams for workpieces that are based on data from experimental studies of linear or flat stress state do not reflect the laws accumulation damage during a volume stress state (when  $I_3(T_\sigma) \neq 0$ ). In [6], the stress state indicator was introduced. This indicator reflects the influence by the third invariant of the tensor or stress deviator:

$$\chi = \frac{\sqrt[3]{I_3(T_\sigma)}}{\sqrt{3I_2(D_\sigma)}} = \frac{\sqrt[3]{\sigma_1\sigma_2\sigma_3}}{\sigma_i} \quad (4)$$

or

$$\chi' = \frac{\sqrt[3]{I_3(D_\sigma)}}{\sqrt{3I_2(D_\sigma)}} = \frac{\sqrt[3]{S_1S_2S_3}}{\sigma_i}, \quad (5)$$

where  $S_1 = \sigma_1 - \sigma$ ,  $S_2 = \sigma_2 - \sigma$ ,  $S_3 = \sigma_3 - \sigma$  – the main deviator of stress tensor,

$$\sigma = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \text{ – average stress.}$$

These plasticity diagrams do not take into account the destruction process by shear, whereas the accumulation of deformations occurs predominantly under by shear. When material destroyed by shear, it is proposed in [7] to present the plasticity diagrams as a function  $e_p = f(\theta)$ , in which the stress state index is defined as:

$$\theta = \frac{1 - k\eta}{\omega}, \quad (6)$$

where  $k$  – the material parameter which is determined experimentally.

For steels of different grades, it can be taken as  $k = 0.05$ ; for aluminum alloys,  $k = 0.1$  [3]. In case of stretching –  $\theta = 1.8$ , shift –  $\theta = \sqrt{3}$ , uniaxial compression –  $\theta = 2.1$ , biaxial stretching –  $\theta = 1.6$ , biaxial compression –  $\theta = 2.4$ .

$$\omega = \frac{\tau_{\max}}{\sigma_i}, \quad (7)$$

where  $\tau_{\max}$  – the maximum tangential tension.

In modeling if material destroyed by break, when the plane of failure is close to the plane on which the maximum normal stresses act in [7], plasticity diagrams are proposed to be represented in the case of a function unified for different stressed states  $\varepsilon_p = f(\beta)$  in which:

$$\beta = \frac{1 - s\eta}{\nu}, \quad (8)$$

where  $\eta$  is determined by the equation (2),

$$\nu = \frac{\sigma_1}{\sigma_i}. \quad (9)$$

Where  $\sigma_1$  – maximum of principal stresses  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ ;  $s$  – the material parameter, which is usually taken equal to  $k$ .

$$\text{In case of stretching: } \sigma_i = \sigma_1; \sigma_2 = \sigma_3 = 0; \beta = \frac{[1 - s(1)]\sigma_i}{\sigma_i} = 1 - s = 0.95.$$

For shear:  $\sigma_1 = \tau$ ;  $\sigma_2 = 0$ ;  $\sigma_3 = -\tau$ ;  $\sigma_i = \sqrt{3}\tau$ ;  $\beta = \sqrt{3}$ .

For compressing:  $\sigma_1 = \sigma_2 = 0$ ;  $\sigma_3 = -\sigma$ ;  $\beta = \frac{[1-s(-1)]\sigma_i}{0} = \infty$ .

For stretching, compression and torsion, the stress state is equal to  $\chi = 0$ .

The use of  $\beta$  is caused by an “anomalous” increase in plasticity under of uniaxial and biaxial stretching. When constructing the plasticity diagrams for steel 20, the data on ultimate strains were obtained: for stretching –  $\varepsilon_p(\eta=1) = 0.6$ ; for flat deformation –  $\varepsilon_p(\eta = 1.73) = 0.4$ ; for biaxial stretching  $\varepsilon_p(\eta = 2) = 0.66$  (fig. 2).

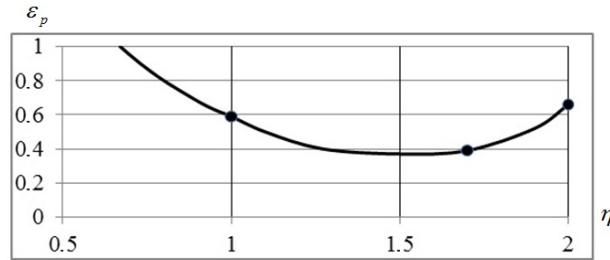


Fig. 2. The plasticity diagram of steel 20

Such results contradict the concept that, with the tightening of the stress state scheme, the plasticity should decrease. G.D. Dell proposes to eliminate this contradiction using the parameter  $\beta$  (8). Then the graph presented in Fig. 3. can be obtained. With an increase in the parameter  $\beta$ , the limiting deformation monotonically increases.

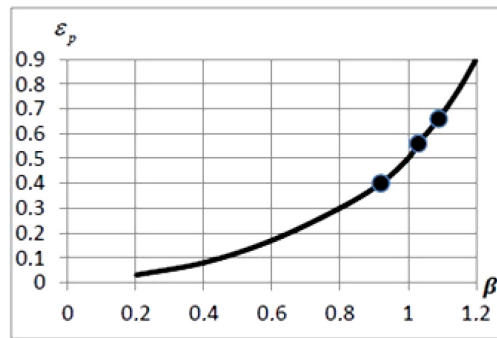


Fig. 3. The plasticity diagram of steel 20

The disadvantage of this approach is the dependence for the parameter  $\beta$  on the material type. The parameter  $s$  in the formula (8) must be determined by experiment.

The “anomalous” plasticity with an increase in  $\eta$  can be explained by the action of two destroy types during stretching of billets from materials prone to local thinning. This is a breaking in the middle of the billet and cutting near the periphery. In addition, the plasticity increase can also be caused by the influence of the third invariant of the stress tensor. Experimental data obtained by materials testing (the steels P12, P18, P9, 40X, 45, P6M5, duralumin) in a high-pressure chamber were presented in [6]. The maximum pressure was 3000 MPa. The experiments were carried out on blanks subjected to torsion together with stretching against the background of hydrostatic pressure. At the same time, deformation methods that ensure the constancy of the stress state parameter ( $\eta = \text{const}$ ) were applied. The connection between the hydrostatic pressure  $q$  and the twisting angle  $\varphi$  must comply with the equation:

$$q = \sigma_i \frac{1 - \eta B}{3B}, \tag{10}$$

where

$$B = \sqrt{1 + \frac{r_0^2 4\pi^2}{3t^2 z}}, \quad (11)$$

$z = \frac{\Delta l}{a_0}$  – lengthening parameter;  $t$  – screw thread pitch-nuts; The axial stroke is provided by rotating the screw at an angle  $\varphi$ .

$$dl_z = \frac{dl}{l} = \frac{dz}{z} = \frac{\frac{t}{2\pi l_0} d\varphi}{1 + \frac{t}{2\pi l_0} \varphi}, \quad (12)$$

where  $z = z_0 + \frac{\varphi t}{2\pi}$ . In experiments  $P + q$ ,  $\frac{dl_z}{dl_i} = 1$ :

$$q = \frac{\sigma_i}{3} (1 - \eta). \quad (13)$$

In the case of experiments  $M + q$ :

$$q = -\frac{\sigma_i}{3} \eta. \quad (14)$$

Accumulated strain rate:

$$\bar{\varepsilon}_i = \int \sqrt{1 + \frac{r_0^2 4\pi^2}{3t^2 z}} \frac{dz}{z}, \quad (15)$$

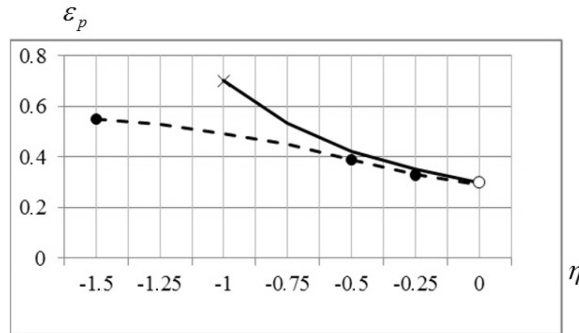
or after integration equation:

$$\bar{\varepsilon}_i = -2B - \ln \left| \frac{1+B}{1-B} \right| + A, \quad (16)$$

where

$$A = 2\sqrt{1 + \frac{r_0^2 4\pi^2}{t^2}} - \ln \frac{1 + \sqrt{1 + \frac{r_0^2 4\pi^2}{3t^2}}}{1 - \sqrt{1 + \frac{r_0^2 4\pi^2}{t^2}}}. \quad (17)$$

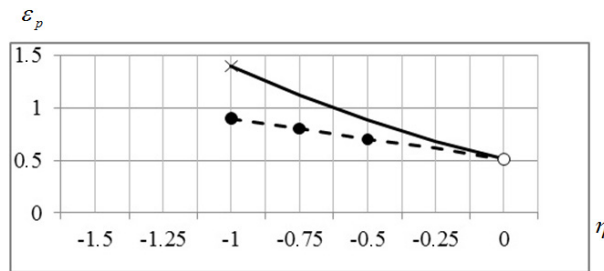
On fig. 4-5 shows the experimental data obtained by testing duralumin and steel P18 according to the programs  $\eta = \text{const}$ ,  $\eta = -0.5$ ,  $\eta = -0.25$  (duralumin) and  $\eta = -1$ ,  $\eta = -0.75$ ,  $\eta = -0.5$  for steel P18.



● – destruction for  $I_3(T_\sigma) \neq 0$ ; ○ – torsion; ✕ – shrinkage; ----  $I_3(T_\sigma) \neq 0$ .

Fig. 4. Effect  $I_3(T_\sigma)$  on ductility (duralumin)

Comparison of the plasticity diagrams constructed for plane and linear stress states allowed us to conclude that the third invariant of the stress tensor suppresses plasticity in the area of  $0 \geq \eta \geq -2$ . Thus, the plasticity diagram is not universal for various stress states. With a decrease in the parameter  $\eta$  on the range of  $0 \geq \eta \geq -1$ , the discrepancy between  $\varepsilon_p I_3(T_\sigma) = 0$  and  $I_3(T_\sigma) \neq 0$  increases.



● – destruction for  $I_3(T_\sigma) \neq 0$ ; ○ – torsion; ✕ – shrinkage; ----  $I_3(T_\sigma) \neq 0$ .

Fig. 5. Effect  $I_3(T_\sigma)$  on ductility (steel P18)

Similar experiments were also carried out in a high-pressure chamber for steels 45, R6M5 and R18. Experiments  $M + q, P + M + q$  ( $1 \geq \eta \geq 0$ ) showed that in the range of the parameter  $1 \geq \eta \geq 0$ , the plasticity is greater with the third invariant of the stress tensor. In fig. 6–8, the plasticity diagrams constructed under the linear and plane stressed states are compared with the diagram at  $\eta = \text{const}$  ( $I_3(T_\sigma) \neq 0$ ).

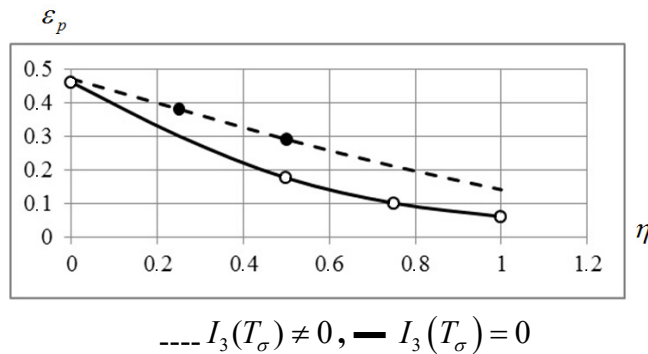


Fig. 6. Effect  $I_3(T_\sigma)$  on ductility (steel P18), experiments  $M + q, R + M + q$

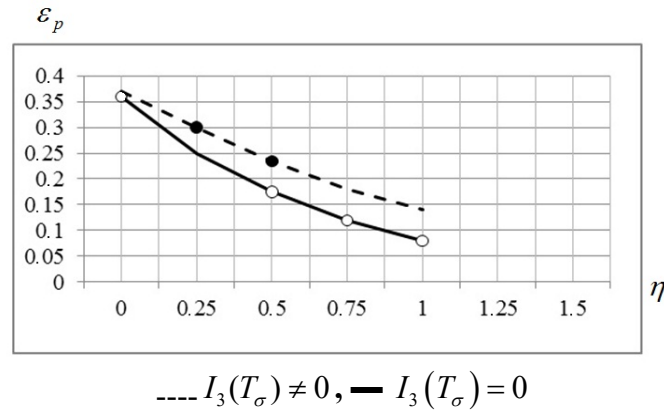


Fig. 7. Effect  $I_3(T_\sigma)$  on ductility (steel P6M5), experiments  $M + q, R + M + q$

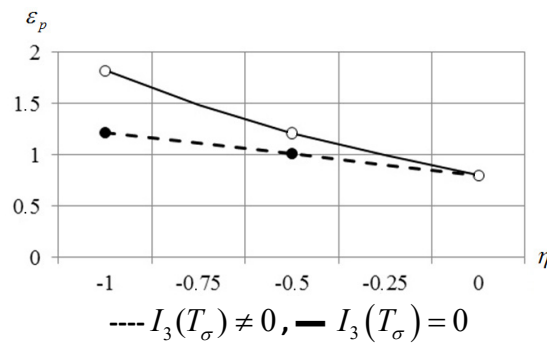


Fig. 8. Effect on ductility (steel 45), experiments  $M + q, R + M + q$

Thus, in the research of technological processes of metal forming, where the volumetric stress state is realized, it is necessary to use the plasticity diagram constructed taking into account the third invariant of the stress tensor. The representation of plasticity diagrams in a three-dimensional surface  $\varepsilon_p = f(\eta, \chi)$  is laborious and often impossible due to the lack of experimental data. In connection with this, a method is proposed which allows the use of ordinary plasticity diagrams by parameter  $\eta$  correction on the  $\varepsilon_p = \varepsilon_p(\eta)$  plasticity diagram with diagrams obtained for the volume stress state at  $\eta = \text{const}$  ( $I_3(T_\sigma) \neq 0$ ). The plasticity in the volume stress state is higher (Fig. 7) compared with the ductility in the plane stress state and with increasing parameter  $\eta$  this difference increases. Thus, the dependence of plasticity on the parameters  $\eta$  and  $\chi$  can be represented using a three-dimensional surface. However, in technical literature, plasticity diagrams are taken to be presented on a plane. In this regard, we describe the stress state index as a function of three invariants of the tensor. The correction of the parameter  $\eta$  in the plasticity diagram ( $\varepsilon_p = \varepsilon_p(\eta)$ ) represents the influence of  $I_3(T_\sigma)$  on plasticity. Stress indicator:

$$\lambda = \frac{I_1(T_\sigma)}{\sqrt{3I_2(D_\sigma)}} \left[ 1 + f \left( \frac{I_1(T_\sigma)}{\sqrt{3I_2(D_\sigma)}} \right) \frac{\sqrt[3]{I_3(T_\sigma)}}{\sqrt{3I_2(D_\sigma)}} \right], \quad (18)$$

where  $f \left( \frac{I_1(T_\sigma)}{\sqrt{3I_2(D_\sigma)}} \right)$  – experimentally defined function. Then equation (18) can be represented as:

$$\lambda = \eta [1 + f(\eta) \chi]. \tag{19}$$

The function  $f(\eta)$  can be represented as a polynomial:

$$f(\eta) = A\eta^2 + B\eta + C, \tag{20}$$

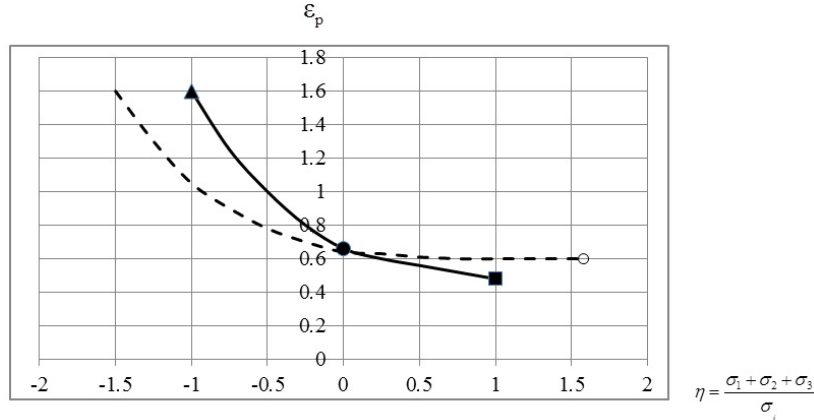
where  $A, B, C$  are the coefficients of the approximating polynomial. The value of the function  $f(\eta)$  is determined from the equation:

$$f(\eta) = \frac{\lambda - \eta}{\eta \chi}. \tag{21}$$

The coefficients  $A, B, C$  of the approximating polynomial turned out to be equal:  $A = -4.1, B = -6.51, C = -6.51$  for steel 40X and  $A = -3.1, B = -5.89, C = -6.44$  for steel 45. Thus, using the plasticity diagrams  $\varepsilon_p = \varepsilon_p(\eta)$  and equation (19), we can estimate the correction associated with the effect of  $I_3(T_\sigma)$  to  $\varepsilon_p$ . Boundary deformation during volume deformation can be defined as the difference of the parameters  $(\eta - \lambda)$ . Consider the method of constructing plasticity diagrams in the area  $0 \leq \eta \leq 2$ , using the third invariant of the stress tensor.

In fig. 9 shows the plasticity diagram constructed using the third invariant of the stress tensor.

If an experimentally constructed plasticity diagram is known, can it be constructed after preliminary plastic deformation for any kind of stress state? This determines the solution of the above problem in the future. Its solution is based on the tensor description of damage accumulation [8].



----  $I_3(T_\sigma) \neq 0$ , —  $I_3(T_\sigma) = 0$ ;  
 ■ – shrinkage ( $\eta = 1$ ); ▲ – compression ( $\eta = -1$ ); ● – torsion ( $\eta = 0$ ); ○ –  $\eta = 1.55$ .

Fig. 9. The plasticity diagram of steel 20

If the plasticity diagram by component of the damage tensor  $\psi_x, \psi_{xy}, \dots$  and the diagram of the accumulated deformation in the plastic deformation determined in the X, Y, Z coordinate system are known, then a plasticity diagram of the deformed metal can be constructed. Let it be required to determine the plasticity of this material under a stressed state, to which the stress state indicators equivalent  $\eta'_1, \eta'_2$ , and the tensor  $\beta'_x, \beta'_{xy}, \dots$ . Then increments of the damage tensor components with additional deformation before destruction will be equal to:

$$\begin{aligned} \Delta \psi_x &= \beta'_x \left[ \phi(\bar{\varepsilon}_0 + \varepsilon'_0, \eta'_1, \eta'_2) - \phi(\bar{\varepsilon}_0, \eta'_1, \eta'_2) \right] \\ \Delta \psi_{xy} &= \beta'_{xy} \left[ \phi(\bar{\varepsilon}_0 + \varepsilon'_0, \eta'_1, \eta'_2) - \phi(\bar{\varepsilon}_0, \eta'_1, \eta'_2) \right] \dots \end{aligned} \tag{22}$$



Destruction process can be represented as equation:

$$(\psi_x + \Delta\psi_x)^2 + (\psi_{xy} + \Delta\psi_{xy})^2 + (\psi_{yx} + \Delta\psi_{yx})^2 + \dots = 1. \quad (23)$$

Or:

$$\Delta\psi_x^2 + \Delta\psi_{xy}^2 + \Delta\psi_{yx}^2 + \dots + 2(\psi_x\Delta\psi_x + \psi_{xy}\Delta\psi_{xy} + \psi_{yx}\Delta\psi_{yx} + \dots) + \psi_0^2 = 1, \quad (24)$$

In equation (24):

$$\psi_0^2 = \psi_x^2 + \psi_{xy}^2 + \psi_{yx}^2 + \dots \quad (25)$$

From the equalities (22), (24) we obtain the quadratic equation, from which we find:

$$\phi(\bar{\varepsilon}_0 + \varepsilon_0', \eta_1', \eta_2') = \phi(\bar{\varepsilon}_0, \eta_1', \eta_2') - D + \sqrt{1 + D^2 - \psi_0^2}, \quad (26)$$

where

$$D = \beta_x'\psi_x + \beta_{xy}'\psi_{xy} + \beta_{yx}'\psi_{yx} + \dots \quad (27)$$

After the approximation, the equation for the plasticity of the deformed metal is obtained:

$$\varepsilon_p' = \varepsilon_p \left[ -\frac{\bar{\varepsilon}_0}{\varepsilon_p} - \frac{1-a}{2a} + \sqrt{\left(\frac{\bar{\varepsilon}_0}{\varepsilon_p} + \frac{1-a}{2a}\right)^2 - \frac{D}{a} + \frac{1}{a}\sqrt{1+a^2-\psi_0^2}} \right]. \quad (28)$$

Here  $\varepsilon_p$  is the plasticity of the undeformed metal under a stressed state with  $\eta_1 = \eta_1'$ ,  $\eta_2 = \eta_2'$ . Parameter  $a$  is the approximation coefficient given in the criterion [7].

$$\psi_{ij} = \int_0^{\varepsilon_i} (1-a + 2a\varepsilon_i^* / \varepsilon_p) \beta_{ij} d\varepsilon_i^* / \varepsilon_p. \quad (29)$$

According to the experimental data given in [8],  $a = 0.5$ .

Thus, using (30), it is possible to calculate the ultimate deformation of a deformed metal for any exponent of the stressed state.

Using phenomenological theories of deformability, in which the accumulation of damage is described by tensor models, it allows us to predict the technological heritage of the material in the form of residual plasticity for finished part.

If during the processing the components of the damage tensor at a given point of the workpiece are  $C$ , then during the subsequent stretching test in the direction of the  $x_1$  axis at a given point, the components of the tensor change by the amount  $\Delta\psi_{ij}$ .

If the destroy equation under such stretching is written as:

$$(\psi_{ij} + \Delta\psi_{ij})(\psi_{ij} + \Delta\psi_{ij}) = 1, \quad (30)$$

then from it it is possible to obtain an equation for the limiting additional deformation of the stretching in the direction to the  $x_1$  axis [9]:

$$\frac{\varepsilon_{p11}}{\varepsilon_p} = -\frac{1}{2} - \frac{\varepsilon_i^*}{\varepsilon_p} + \sqrt{\left(\frac{1}{2} + \frac{\varepsilon_i^*}{\varepsilon_p}\right)^2 - \sqrt{6}\psi_{11} + \sqrt{6\psi_{11}^2 + 4(1 - \psi_{ij}\psi_{ji})}}, \quad (31)$$

where  $\varepsilon_i^*$  is the accumulated deformation during the forming of the workpiece;  $\varepsilon_p = \varepsilon_p(\eta=1)$  is plasticity of the metal at  $\eta=1$ ;  $\varepsilon_{p11}$  is residual plasticity in stretching to the direction 11.

Since  $\psi_{11}$  depends on the direction to  $x_1$ , the residual plasticity of  $\varepsilon_{p11}$  also turns out to depend according to direction. Thus, using eq. (33), one can estimate the plasticity anisotropy in any zone of branching on pipe, which is produced by the method of cold plastic deformation.

As an example, which illustrates the practical significance of this research, we can give to evaluate the plasticity for bends, which is produced by the cold plastic deformation method on combined scheme, which includes deforming stretching of the billet. In this case, the billet in the pipe form undergoes plastic bending followed by loss of plastic deformation stability.

In [10] the equation (18) obtained on the basis of the tensor representation about the accumulation of damages is presented, which makes it possible to estimate the residual ductility of the finished bend.

$$\frac{\varepsilon_p}{\delta_p} = \frac{D_1 b \varepsilon_i^*}{200 \delta_p} + \frac{D_1 b}{100} \sqrt{\left(\frac{1}{2} + \frac{100\varepsilon_i^*}{D_1 b \delta_p}\right)^2 + \sqrt{2} \left[ \psi_s - \psi_\theta + \sqrt{(\psi_s - \psi_\theta)^2 + (1 - \psi_{ij}\psi_{ji})} \right]}, \quad (32)$$

where  $\delta_p$  – elongation at break,  $b = \frac{C_1}{B_1}$  ( $C_1 = 1.03$ ,  $B_1 = 0.55$  for steel 20;  $C_1 = 1.08$ ,  $B_1 = 0.67$  for steel X18H9T),  $D_1 = 0.66$  for steel 20;  $D_1 = 0.73$  for steel X18H9T.

$$\begin{aligned} \psi_\alpha = \psi_\theta &= \frac{1}{4} \sqrt{\frac{2}{3}} \cdot \left[ \frac{\varepsilon_i}{\varepsilon_{p(\eta=2)}} + \left( \frac{\varepsilon_i}{\varepsilon_{p(\eta=2)}} \right)^2 \right]; \\ \psi_s &= -(\psi_\alpha - \psi_\theta) = -2\psi_\alpha; \\ \psi_{ij}\psi_{ji} &= \frac{1}{4} \left[ \frac{\varepsilon_i}{\varepsilon_{p(\eta=2)}} + \left( \frac{\varepsilon_i}{\varepsilon_{p(\eta=2)}} \right)^2 \right]^2. \end{aligned} \quad (33)$$

In this research, an experimental verification to theoretical research of the residual plasticity for billets, previously deformed before obtain finished products, is carried out.

From the outer zone of the bends  $90^\circ 57 \times 4$ ;  $90^\circ 89 \times 4.5$ ; made of steel 20 ( $\delta P = 30\%$ ), flat samples were cut out. For the tensile test to longitudinal and circumferential directions of the steeply bend these samples were used. The stretching of these samples showed that their residual elongations to longitudinal and circumferential directions are approximately the same. The table 1 shows the theoretical and experimental results of residual plasticity.

One of the most important characteristics bends which made by the cold plastic deformation method is their residual plasticity under working loads (especially under cyclic pressure loads at elevated temperatures). This plasticity is estimated approximately by the stretching test results of samples cut from the bends to different directions.

Table 1. Comparison by residual plasticity (theoretical and experimental results)

Size of bend	Theoretical results				experimental results		Divergence
	$\psi$	$\epsilon_{PS}$	$\epsilon_{PPas}$	$\epsilon_{Pa} = \epsilon_{P\theta}$	$\epsilon_{Pa}$	$\epsilon_{P\theta}$	%
90° 57×4	0.35	0.58	0.27	0.43	0.41	0.37	9.3
90° 89×4.5	0.4	0.59	0.25	0.41	0.38	0.35	14.6

## Conclusions

1. The calculation method for determining the plasticity of pre-deformed metals during their processing by pressure has been developed. This method is based on a fracture model, which in turn is based on the tensor description of damage accumulation. With known mechanical characteristics, as well as with known plasticity diagrams, the fracture model makes it possible to evaluate the plasticity of pre-deformed bend for any kind of stress state.

2. When manufacturing steeply curved branches using the pipe extrusion method, the procedure was tested. Verification of the mathematical model has shown a high level of its adequacy, and it can be used in assessing the plasticity of pre-deformed billet.

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